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Andrew G. Lyne

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By Andrew G. Lyne

University of Manchester, Jodrell Bank Observatory,
Macclesfield, Cheshire SK11 9DL, UK

The magnetic fields of young neutron stars are the most intense known to humankind, exceeding those achievable on the Earth by perhaps six orders of magnitude. It is these magnetic fields which make neutron stars observable as radio pulsars. In this paper we discuss what we know about these fields and how they evolve with time. The fields of the oldest pulsars, the millisecond pulsars, are much smaller and appear to be reduced during the process of accretion which is responsible for their rapid rotation rates. The pulsed nature of the radiation from pulsars also makes them excellent probes of the very weak field of the interstellar medium and we briefly describe what we have learned about these fields, which are perhaps 20 orders of magnitude smaller than the fields in the neutron stars themselves.

Keywords: neutron stars; pulsars; magnetic fields; magnetars

1. Introduction

Neutron stars are widely believed to be formed in the angular-momentum-conserving supernova collapse of the cores of massive stars ($M > 7M_{\odot}$). The result is a rapidly rotating neutron star of roughly nuclear density whose existence was predicted by Baade & Zwicky (1934) and whose magnetic properties were predicted by Pacini (1967, 1968). The most visible manifestation of neutron stars is during their generally rather short phases as radio pulsars, the first of which was discovered by Hewish *et al.* (1968). While over 1000 of these are now known, many neutron stars are now observed at X-ray wavelengths and higher energies. It is believed that the radio emission originates in electrodynamic processes in the open magnetic field lines of the magnetosphere above the magnetic poles. The result is a large flux of highly relativistic charged particles which radiate coherently to produce a beam of radiation along the magnetic field lines, which is observed as pulses as the rotation of the star passes the beam over the Earth.

These pulses allow us to detect the objects and to observe their rotation with very high precision. Simple electrodynamic arguments allow the estimation of the surface magnetic-field strength from the rate of slow-down.

2. The magnetic field

What do we know of this magnetic field and its evolution? Most pulsars slow down and fade on time-scales of only a few million years although there is a separate population, the millisecond pulsars, which evolve from a subset of these normal pulsars and whose lifetimes are measured in billions of years. The main instrument of the slow-down in both cases is the magnetic field of the star. Firstly, its form seems to be rather simple and is probably dipolar as judged from the high degree of linear polarization, smooth variation through the pulse in the position angle of the linear polarization and, in a few pulsars when the geometry is favourable, the observation of two pulses *ca.* 180° apart in rotational phase, corresponding to the two magnetic poles (see, for example, Lyne & Manchester 1988).

The magnitude of the magnetic field is estimated using the assumption that the main energy loss is in the form of electromagnetic radiation from a rotating dipole *in vacuo*. For a generalized spin-down, the braking torque is proportional to some power n of the rotation rate ν, so that $\dot{\nu} \propto -K\nu^n$. In the case of magnetic dipole braking, the rate of loss of angular momentum is equal to the magnetic torque (Ostriker & Gunn 1969):

$$2\pi\dot{\nu}I = -\frac{16M^2\pi^3}{3c^3}\nu^3, \tag{2.1}$$

where M is the component of magnetic dipole moment normal to the rotation axis and I is the moment of inertia of the neutron star. In this case, clearly, $n = 3$, leading to an estimate of the surface magnetic field of

$$B \approx \frac{M}{R^3} = \sqrt{\frac{3c^3I\dot{\nu}}{8\pi^2R^6\nu^3}} = 3.2 \times 10^{19}\sqrt{P\dot{P}}\ \mathrm{G}, \tag{2.2}$$

where canonical values for the stellar radius R and moment of inertia I are taken to be 10 km and 10^{45} g cm^2. These values depend somewhat upon uncertainties in the equation of state of matter in the core of the neutron star, but do not change greatly over the range of uncertainty. The same estimate for the field is obtained for an aligned rotator, where the energy loss is in the form of charged particles from the open field lines (Goldreich & Julian 1969).

This simple model also provides an estimate of the age, obtained by integrating the generalized spin-down equation and assuming that the pulsar started out life rotating much faster than at present. Then

$$\tau = -\frac{\nu}{(n-1)\dot{\nu}} = -\frac{\nu}{2\dot{\nu}} = \frac{P}{2\dot{P}}, \tag{2.3}$$

where the latter two expressions are for the case of $n = 3$ and give the ‘characteristic age’.

The magnetic fields of pulsars calculated using equation (2.2) range between *ca.* 10^8 and 10^{13} G. There is also some independent evidence for such large fields from the detection of cyclotron lines in the spectra of some X-ray burst sources, indicating fields of order 10^{12} G on the surface of accreting neutron stars.

The pulsar magnetic fields are shown in figure 1, in which they are plotted against rotational period P. In this diagram we expect pulsars to be formed towards the top left-hand corner and to move horizontally to the right as they spin down. Indeed, about a dozen of the youngest pulsars still have the remnants of the supernova explosion of their birth surrounding them, as indicated by the stars. It is also notable that the main population of pulsars, those in the top right-hand quadrant, are solitary, unlike the predominantly binary nature of their expected progenitors. This clearly arises from velocity ‘kicks’ of several hundred kilometres per second imparted to the

Table 1. *The observed braking indices of five pulsars*

pulsar	n	reference
B0531+21	2.51 ± 0.01	Lyne *et al.* (1993)
B0540−69	2.24 ± 0.04	Boyd *et al.* (1995)
B0833−45	1.4 ± 0.2	Lyne *et al.* (1996)
J1119−6127	3.0 ± 0.1	Camilo *et al.* (2000)
B1509−58	2.837 ± 0.001	Kaspi *et al.* (1994)

neutron stars at birth in the violence of the supernova explosions, resulting in the disruption of most binary systems. Proper motion measurements confirm the high velocity of the population and confirm the consistency of this picture.

However, if the magnetic fields of young pulsars in supernovae are constant, figure 1 suggests that they are not the progenitors of the normal population. In fact, the situation is more stark than this.

For a small number of young pulsars, it is possible to check the value of the braking index n. Differentiation of the generalized spin-down equation leads to

$$n = \nu\ddot{\nu}/\dot{\nu}^2. \tag{2.4}$$

Thus, determination of the frequency second derivative permits a measurement of n. Table 1 presents the five measurements of braking indices which have been possible to date. The existence of timing noise and glitches has precluded the determination of any other reliable values. It is clear that four of the values of n are significantly less than 3, indicating that, for these young pulsars at least, at least one assumption in the theory outlined above is not valid. One possibility is that the magnetic field is not perfectly dipolar. This may be due to the intrinsic stellar field being non-dipolar, or due to charged particle flow along the open field lines which may distort the configuration of the field. Alternatively, the effective magnetic moment M may be increasing, or the moment of inertia I decreasing, with time. Whichever of these explanations is correct, the values of B calculated using equation (2.2) are increasing with time and these four pulsars are all moving on upward-sloping tracks in figure 1. These are thus heading for a region of the diagram above the main population. It is clear that if the inferred magnetic fields of these pulsars do not subsequently start to decrease, they are not the progenitors of the normal population of pulsars, in conflict with much circumstantial evidence and widespread belief that they are.

The main body of the normal population of pulsars is certainly moving towards the right-hand side of the diagram, and may also be moving downwards if magnetic-field decay is occurring. However, there is no direct observational evidence for this and there are theoretical arguments against it because of the high conductivity of the superconducting star. As the rotation rate decreases, the electrodynamic acceleration and emission processes in the magnetosphere become less effective. The radio luminosity of pulsars is then expected to decrease, resulting in the falling observed density of pulsars to the right-hand edge of the 'island' of normal pulsars and a massive population of dead neutron stars beyond it.

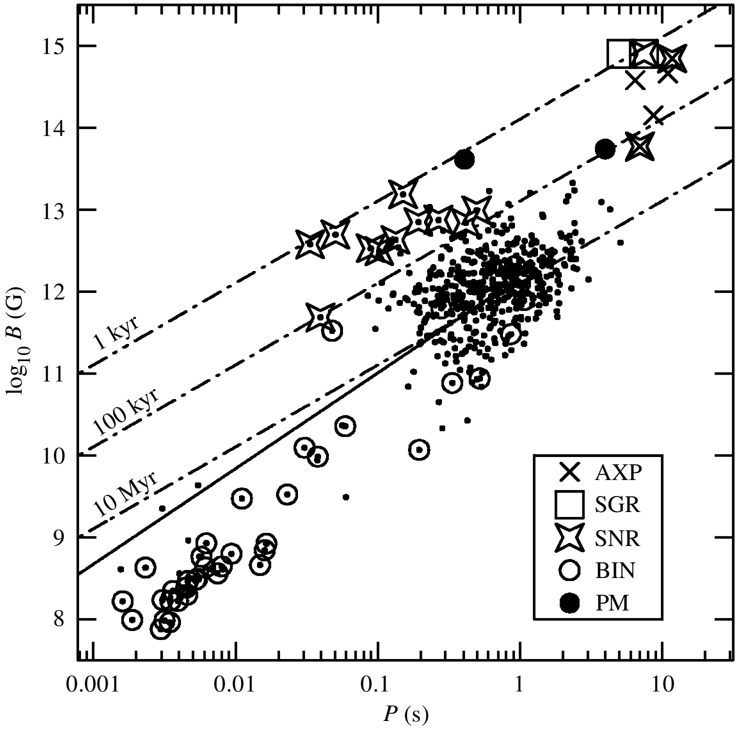

Figure 1. The pulsar population shown on a plot of the surface magnetic field B against the rotational period P. Pulsars with constant field are expected to move horizontally from the left across the diagram, at a rate indicated by the dashed lines of characteristic age. The solid line is the spin-up line, representing the minimum spin-up period possible for a given value of magnetic field.

3. The binary and millisecond pulsars

A few pulsars are in binary systems which must have survived the supernova explosions of their formation, and these can be seen in the body of the main population. It is believed that some of these are the progenitors of the millisecond pulsars seen in the lower left-hand quadrant of figure 1. The manner in which they get to this position is somewhat unclear, although there is a wide concensus concerning the general processes involved, as follows.

A pulsar which is formed in a surviving binary system will undergo the normal lifespan of a solitary pulsar, probably ending up on the right-hand side of figure 1. Eventually, the companion star leaves the main sequence and moves into the 'giant' phase. Accretion of material onto the neutron star from the bloated atmosphere of the companion will result in the formation of an accretion disc and the transfer of orbital angular momentum to the spin angular momentum of the neutron star, moving the star towards the left. This accretion phase is believed to be witnessed through the copious emission of X-rays from the hot gas of the accretion disc. If the companion star is massive enough ($M > 7M_{\odot}$), the core will very shortly undergo supernova collapse after little accretion and spin-up has occurred. Should the binary system survive this second violent event, a highly eccentric double-neutron-star system will be formed, similar to that containing the Hulse–Taylor pulsar PSR B1913+16. There are half-a-dozen such systems presently known.

In the case of a low-mass companion, the accretion phase will be protracted and result in a very rapidly rotating neutron star. If the accretion is sustained for long enough, the spin-period of the pulsar will reach a minimum value, P_{min}, which is determined by the value of the magnetic field which prevents further spin-up (see Bhattacharya & van den Heuvel (1991) and references therein):

$$P_{\mathrm{min}} \approx 1.9 \times (B/10^9)^{6/7}\ \mathrm{ms}, \tag{3.1}$$

It is not clear how the magnetic fields of these objects reach the low values of 10^8 or 10^9 G. Field decay prior to the spin-up is one possibility, although a more favoured one is the quenching of the field during the accretion process, in which the magnetic-field lines are buried below the accreted matter. The reduction of the field in this way would permit further spin-up, and the pulsar would slide down the 'spin-up' line defined by the equation above.

Once the giant phase of the companion is over, accretion ceases and a rapidly spinning neutron star with a small magnetic field remains, usually in orbit with the white-dwarf remnant of the core of the companion star. Such a millisecond pulsar has a characteristic, or spin-down, age which typically exceeds 10^9 years.

4. The magnetars

The past few years have seen the identification of a class of pulsating high-energy sources whose rate of spin-down suggests that they might be neutron stars possessing even greater magnetic fields than the 10^{13} G of the radio pulsars (Thompson & Duncan 1996; Heyl & Hernquist 1997). These are the so-called magnetars which fall into two classes, the 'anomalous X-ray pulsars' (AXPs) and the 'soft gamma-ray repeaters' (SGRs), none of which have been detected at radio wavelengths. The AXPs are characterized by X-ray periods in the range 5–12 s and extremely rapid spin-down (see, for example, Mereghetti & Stella 1995; Gotthelf & Vasisht 1998), while the SGRs exhibit occasional enormous bursts of gamma radiation and AXP-like X-ray pulsations during quiescence. Four of the objects are clearly associated with supernova remnants. For at least some of the sources, there are no observed Doppler variations in the pulsation periods which might be due to binary motion, indicating that they are solitary neutron stars.

The period derivatives of five AXPs and two SGRs suggest that they have surface magnetic fields of $10^{14} < B < 10^{15}$ G. Their most remarkable property is that the X-ray luminosity is several orders of magnitude greater than the energy loss from the spin-down, which powers normal radio pulsars. Unlike other X-ray sources, accretion cannot be the source of the energy, since there is no companion star. It has been suggested that the energy must come from either the thermal energy of the newly formed neutron star or the energy stored in the intense magnetic field.

It has been suggested that the magnetars form a separate population from the normal pulsars. Until recently, there has been a clear gap between the normal population and the magnetars (figure 1).

However, a new survey for radio pulsars has a bearing on this matter. The pulsar multibeam survey of the galactic plane using the 64 m Parkes radio telescope is conducted at the relatively high radio frequency of *ca.* 1400 MHz (see, for example, Lyne *et al.* 2000). This survey is not prone to propagation effects in the interstellar medium and high background temperatures which have compromised many of the earlier surveys at lower frequencies. The survey has already discovered more than 400 new pulsars and is expected to more than double the total of 700 previously known. Among the newly discovered pulsars are two radio pulsars with magnetic fields of 4.1×10^{13} and 5.5×10^{13} G (Camilo *et al.* 2000), values which are intermediate between those of the strongest-field radio pulsars and the magnetars. The larger-field object is certainly not a bright X-ray source (Pivovaroff *et al.* 2000).

These new pulsar multibeam (PM) discoveries, indicated by the large filled circles in figure 1, now occupy the former gap and there is no clear distinction between the two populations in this diagram. Both sets of objects seem to have had a recent origin in supernova events. The radio pulsars and magnetars may only be distinct in the energy of their electromagnetic emission, either radio or X-ray. At present, there is no obvious evolutionary relationship and the reason for this difference is not clear.

5. The galactic magnetic field

Radio pulsars make superb probes of the galactic magnetic field because of three unique properties. Firstly, the radiation is usually highly linearly polarized, allowing the measurement of the Faraday rotation of the position angle of the linear polarization. At wavelength λ, the rotation $\Delta\psi$ gives an estimate of the rotation measure (RM), which is related to the electron density n_{e} and longitudinal component of the magnetic field $B_{\|}$ along the line of sight to the pulsar:

$$\mathrm{RM} = \frac{\Delta\psi}{\lambda^2}\ (\mathrm{rad\ m^{-2}}) = 0.81 \int n_{\mathrm{e}} B_{\|}\, \mathrm{d}l\ (\mathrm{\mu G\ pc\ cm^{-3}}). \tag{5.1}$$

Secondly, the pulsed nature of the radiation allows the determination of the dispersion measure (DM), which is related just to the electron density n_{e} along the line of sight:

$$\mathrm{DM} = 0.000\,12 \frac{\mathrm{d}\tau}{\mathrm{d}\nu}\nu^3 = \int n_{\mathrm{e}}\, \mathrm{d}l\ (\mathrm{pc\ cm^{-3}}), \tag{5.2}$$

where $\mathrm{d}\tau/\mathrm{d}\nu$ is the dispersion in pulse arrival time with frequency.

Both of these quantities have been measured for over 300 pulsars and their ratio gives a direct estimate of the mean component of the magnetic field along the line-of-sight:

$$\langle B_{\|}\rangle = 1.232 \frac{\mathrm{RM}}{\mathrm{DM}}\ (\mathrm{\mu G}). \tag{5.3}$$

Finally, the known pulsars are widely distributed throughout the Galaxy, albeit much more densely within a few kiloparsecs of the position of the Sun. In principle, with a number of pulsars at different distances along a particular radial direction, for two adjacent pulsars, the differences in RM and DM allow a direct measurement of the local value of $B_{\|}$:

$$B_{\|} = 1.232 \frac{\Delta\mathrm{RM}}{\Delta\mathrm{DM}}\ (\mathrm{\mu G}). \tag{5.4}$$

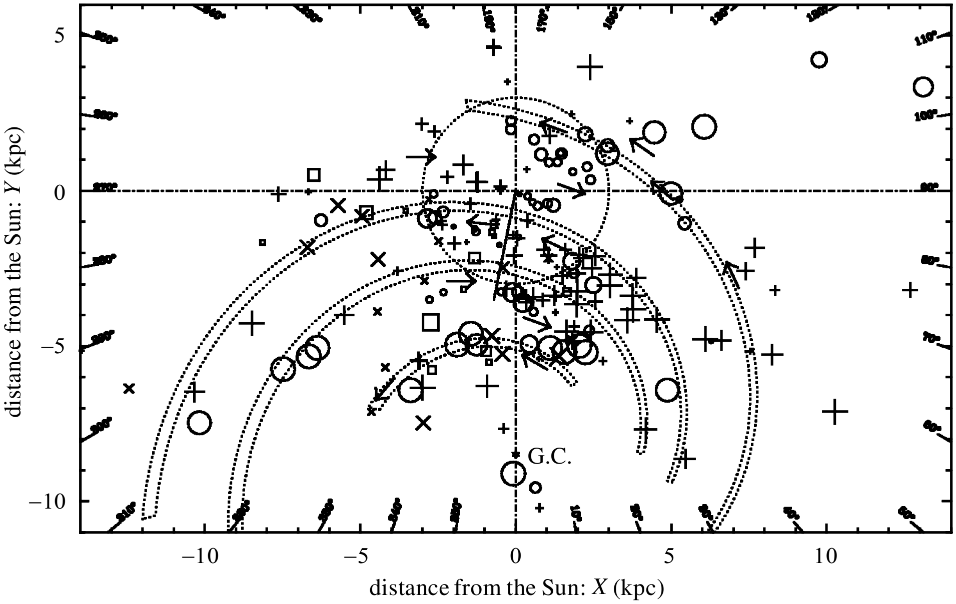

Figure 2. The RM distribution of pulsars with $b < 8^\circ$ projected onto the galactic plane (after Han *et al*. 1999). The size of the symbols is proportional to the square root of the RM, with positive RMs represented by a cross and negative RMs by a circle. New measurements are indicated by × and open squares. The directions of the prevailing magnetic field, as revealed by these observations, are indicated by arrows. The approximate locations of four spiral arms are indicated.

Since the DM, combined with a model for the free-electron distribution through the Galaxy, gives an estimate of distance, we can in principle build up a model of the galactic magnetic field. A number of authors have conducted such studies (Lyne & Smith 1989; Rand & Kulkarni 1989; Rand & Lyne 1994; Han *et al*. 1997, 1999).

While there is some difficulty in representing the three-dimensional nature of the available data, figure 2 gives a rough indication of the large-scale distribution of the data. This shows a plot of rotation measures for pulsars in the galactic plane ($|b| < 8^\circ$), as viewed from above the plane of the Galaxy (after Han *et al*. 1999). The local value of the gradient of the rotation measure gives the longitudinal component of the magnetic field. Such measurements allow a number of conclusions.

(1) The local magnetic field, within a few hundred pc of the Sun, has a magnitude of $B \sim 2\,\mu\mathrm{G}$ towards $l = 90^\circ$.

(2) The field can be seen to reverse direction *ca.* 1 kpc inside the solar circle.

(3) At low galactic latitudes, there is good evidence that a second reversal exists *ca.* 3–4 kpc inside the solar circle.

(4) Towards the inner Galaxy, the field increases in magnitude, reaching 6–7 μG inside the second reversal.

(5) Irregularities are of the same order of magnitude as the organized field, having scale of 50–200 pc.

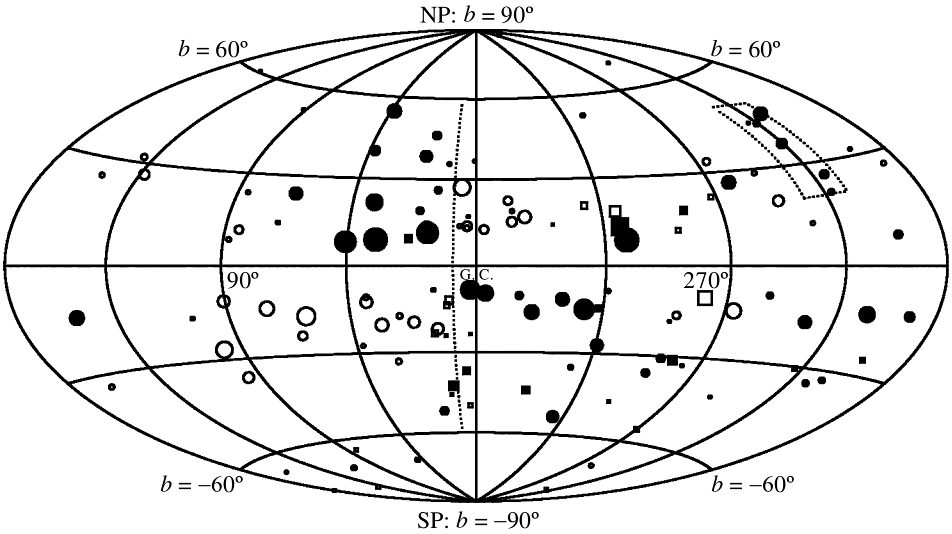

Figure 3. An aitoff projection of the galactic distribution of RMs for pulsars with $b > 8^\circ$ (after Han *et al.* 1999). Filled symbols represent positive RMs, open symbols negative RMs, and their area is proportional to $|\mathrm{RM}|$. The dotted line indicates the antisymmetry with respect to $l = 8^\circ$.

(6) At intermediate latitudes (figure 3), there is evidence from pulsar studies and from the rotation measures of extragalactic sources for a large-scale antisymmetric structure. The field has opposite approximately tangential directions above and below the galactic plane (Han *et al.* 1997, 1999).

There is clearly significant coherent structure in the magnetic field which is reasonably well resolved in the solar neighbourhood. However, beyond a few kiloparsecs, the space density of pulsars falls off rapidly and is insufficient to delineate any detail in the configuration of the field. For the future, we expect that the new high-frequency surveys will increase the density of known pulsars in the galactic plane by a factor of between five and ten and will permit a much fuller three-dimensional study of the magnetic field to further its understanding.

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Discussion

R. Beck (*Max-Planck-Institut für Radioastronomie, Bonn, Germany*). The recent pulsar RMs obtained at the Parkes telescope by Han & Manchester seem not to fit into the large-scale spiral magnetic field with few reversals. What is your interpretation?

A. Lyne. Since the magnetic field within a few kiloparsecs of the Sun does show clear large-scale structure and reversals, it would be surprising if this were not the case elsewhere across the Galaxy. The failure to see this is due in my view to the low space density of detectable pulsars in the distant parts of the Galaxy and any structures are difficult to discern because of under-sampling. The new survey will go a long way to remedying this limitation by increasing the space density of pulsars by an order of magnitude.

D. Lynden-Bell (*Institute of Astronomy, Cambridge, UK*). Is there any indication that there is an increase of magnetic moment associated with glitches?

A. Lyne. In the case of one pulsar, the crab, indeed there is a permanent increase in slow-down rate, consistent with an increase in magnetic field. For other pulsars, any such increase in rotation rate is too small to be detected.

R. Beck. The determination of the regular field strength $B_{||}$ from pulsar RM and DM via RM/DM is valid only if electron density n_e and B are *not correlated* so that $\langle n_e B_{||}\rangle = \langle n_e\rangle\langle B_{||}\rangle$. However, if n_e and $B_{||}$ are *correlated* (or anticorrelated), e.g. $n_e \propto B_{||}$, we find

$$\frac{\mathrm{RM}}{\mathrm{DM}} = K\left(\langle B_{||}\rangle + \frac{\delta n_e^2}{\langle n_e\rangle}\right)^*,$$

where δn_e are the fluctuations of n_e along the line of sight. Could this effect modify the field strength estimates?

A. Lyne. I believe that what I have said is rigorously correct. However, what you say does underline the fact that the mean value of magnetic field derived is weighted by the electron density, so that the field you measure is that where most electrons are.

M. Ruderman (*Columbia University, USA*). Estimates of purely ohmic decay of crustal screening currents which might temporarily keep a core magnetic field buried beneath the surface of a pulsar give decay time-scales more than a hundred times too long to account for observed spin-down indices.

A. Lyne. It's a big problem.

A. Shukurov (*University of Newcastle, UK*). How accurate are pulsar distances obtained from dispersion measures?

A. Lyne. The errors are probably about a few tens of per cent.

F. Graham-Smith (*Jodrell Bank Observatory, University of Manchester, UK*). The magnetic fields of the young pulsars are changing very rapidly. A mechanism which would double the field within the lifetime (i.e. the time-scales of rotation slow down) is necessary. Is there a mechanism for this?

A. Lyne. I do not know of an entirely successful mechanism.

L. Mestel (*The University of New Hampshire, USA*). The spin-down of the neutron star is determined by the amount of magnetic flux emerging from the stellar surface, along which gas can flow to infinity. Perhaps the rapidly rotating pulsars begin with much of the flux beneath the surface. Ohmic decay of currents in the crust will convert the field in the crust into a curl-free structure causing outward diffusion and enabling the observed field initially to increase. The crucial question is of course the time-scale.

A. Lyne. Yes, I believe that is a possibility. What I didn't say was that these fields are believed to be sustained by the superconducting properties of the remnant charged particles within the neutron star. There is another possibility, which was suggested a few years ago by Roger Blandford. Have his views on the thermoelectric enhancement of fields changed during the last 15 years or so?

R. D. Blandford (*Caltech, Pasadena, USA*). No, we actually wrote a paper with mechanisms which I thought initially would work, but when we actually used the best estimates of the transport properties, it failed to produce growth. There were some secondary mechanisms which might still work but they weren't quite as promising for the fields and pulsars, so, in fact, I don't think it ever worked with the rate of growth beating the decay. I agree with Leon (Mestel), actually; I think a good way to do it is to make a pulsar with subsurface fields that aren't too far below the surface so they can emerge through ohmic decay. You only have to double or treble the field—that may be one way of doing it.